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«INVESTIGATION OF ELASTIC PROPERTIES OF SOLIDS AND BIOLOGICAL OBJECTS»

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METHODOLOGICAL INSTRUCTIONS FOR LABORATORY WORK NO.2(1)

«INVESTIGATION OF ELASTIC PROPERTIES OF SOLIDS AND BIOLOGICAL OBJECTS»

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The tutorial outlines a methodology for carrying out the laboratory work devoted to the study of physical laws that describe the deformation processes of different physical nature. The proposed method is based on an experimental study of the elastic modules of different solids and biological materials with help of the clock-type micrometer.

The publication gives a brief description of the theoretical data, which are used to describe the phenomenon that is being studied in the laboratory work. The theoretical calculations which are necessary for the interpretation of the corresponding experimental data and confirmation of the known theoretical laws are given. Also the step-by-step instruction for conducting experimental measurements, and corresponding calculations of physical quantities is presented as well as a brief instructions for operation with the measurement devices.

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Laboratory work № 2(1)

Investigation of elastic properties of solids and biological objects

The aim of the work: to acquaint with the physics of bodies' deformation processes of different physical nature, to study Hooke's law, to measure the elastic modulus of different solids and biological materials.

Equipment: the device for studying the elastic properties of materials, a steel sample and samples of other materials in the form of plates, scales, a ruler, a calliper.

Brief theoretical information.

Such processes that take place in human body as the functioning of the musculoskeletal system, the processes of deformation of tissues and cells, the propagation of waves of elastic deformation, muscle contraction and relaxation, the movement of liquid and gaseous biological media are associated with various mechanical phenomena. For such areas of medicine as the study of the musculoskeletal system, surgery, orthopedics and prosthetics, it is important to know the elastic properties of body tissues and especially bone tissue.

The most important mechanical properties of materials are:

- **elasticity** - the ability of bodies to restore their size (shape and volume) after the stress is removed;
- **flexibility** - the ability of the material to change size under the action of external stress;
- **stiffness** - the ability of the material to resist to external stress;
- **strength** - the ability of bodies to resist destruction under the action of external forces;
- **plasticity** - the ability of bodies to maintain (fully or partially) a change of size after the stress is removed;
- **brittleness** - the ability of the material to collapse without the appearance of noticeable residual deformations;
- **viscosity** - a dynamic property that characterizes the body's ability to resist changes of its shape under the action of tangential stress;
- **fluidity** - a dynamic property of the medium, which characterizes the ability of its individual layers to move with some velocity relative to other layers of this medium.

The change of the relative position of the points of the body, which leads to a change of its shape and size, is called **deformation**. Deformations can be caused by external actions (mechanical, electrical, magnetic) or by changes of temperature of the body.

Let us consider the deformations that occur when forces act on the body.

If the deformation disappears after the cessation of the force action on solids, then deformation is called **elastic**. In this case the particles of the solid return to their original equilibrium position and restore the original size and volume of the body after shifting in the process of deformation. If the deformation persists after the cessation of external action, it is called **plastic** deformation. The intermediate case, ie partial disappearance of deformation, is called **elastic-plastic** deformation.

There are forces that attempt to restore the original size of the body in case of any type of the processes of deformation. These forces are called **elastic forces**.

The elastic force $\vec{F}_{el}$ is the force that occurs during the deformation of the body, it is opposite directed to the direction of displacement of body particles during this deformation.

The question arises: what is the nature of the elastic forces?

All bodies consist of atoms. Atoms are positively charged nuclei around which electrons revolve. All bodies are neutrally charged because both types of charges in the body are balanced under the normal conditions. There is a strong electrical interaction (repulsive and attractive forces) between atoms inside the body. The absolute values of these forces depend on the distance between the atoms. The forces of attraction between the molecules are compensated by the repulsive forces between them at a distance approximately equal to the diameter of the molecule, so the resultant force is zero. The distance between the atoms increases when the body is stretched and the forces of attraction become greater (by absolute values) than the forces of repulsion. The arisen forces of attraction prevent the body from stretching. In case of the reverse process compression, the repulsive forces between the atoms prevail. They prevent the deformation of the body.

The considered mechanism of deformation allows one to make a conclusion that elastic forces have electromagnetic nature.

The following physical quantities are used for the quantitative characterization of the deformation of a body:

1) **absolute deformation** ΔX - if some quantity that characterizes the size or shape of the body (for example, length or volume), takes the value of X due to the deformation of a body, then the change of this quantity under the action of applied force is called absolute deformation:

$$\Delta X = X - X_0 \ ; \tag{2.1}$$

2) **relative deformation** ε - this quantity shows the ratio of the absolute deformation to the initial value of X_0:

$$\varepsilon = \Delta X / X_0 \ . \tag{2.2}$$

Most bodies show elastic properties in case of small relative deformation ($|\Delta X| << X_0$);

3) **mechanical stress** σ - this physical quantity is numerically equal to the elastic force dF_{el} per unit cross-sectional area of the body dS:

$$\sigma = dF_{el}/dS \tag{2.3}$$

The mechanical stress for the linear process is determined as follows:

$$\sigma = \Delta F_{el}/\Delta S. \tag{2.4}$$

If the stress is constant over the entire area, then

$$\sigma = F_{el}/S \ . \tag{2.4a}$$

The stress is said to be normal if the force ΔF_{el} is perpendicular to the cross-sectional area ΔS, and stress is tangent if ΔF_{el} is directed tangentially to ΔS.

It is experimentally established that in the case of elastic deformations **Hooke's law is fulfilled**: **the mechanical stress σ is directly proportional to the relative deformation (elongation) ε of the body:**

$$\sigma = E\,|\,\varepsilon\,|. \quad (2.5)$$

The coefficient of proportionality E is called the elastic modulus or Young's modulus.

Taking into account (2.2) and (2.4a), we can rewrite this law in the following form:

$$F_{el}/S = E\,|\,\Delta X/\,X_0\,|. \quad (2.6)$$

If the relative elongation $\varepsilon = 1$, then $\sigma = E$, so the Young's modulus is equal to the stress that occurs in the rod at its relative elongation equal to 1. From (2.2) it follows that for $\varepsilon = 1$, we will obtain $\Delta X = X_0$, and this means that the Young's modulus is equal to the stress that occurs in the body when the length of the sample is doubled.

In practice the bodies of most of materials (except, for example, rubber) can not double its length under the elastic deformation. The body will break before its length will be doubled. The larger the Young's modulus E is, the less the body is deformed (if other parameters are constant). Thus, the Young's modulus characterizes the resistance of the material to elastic deformation, ie it characterizes the elastic properties of the body.

The experimentally obtained dependence of the stress that occurs in the sample on the relative deformation is called the **deformation diagram**. The deformation diagram for the strain deformation of a metal rod is shown in figure 1.

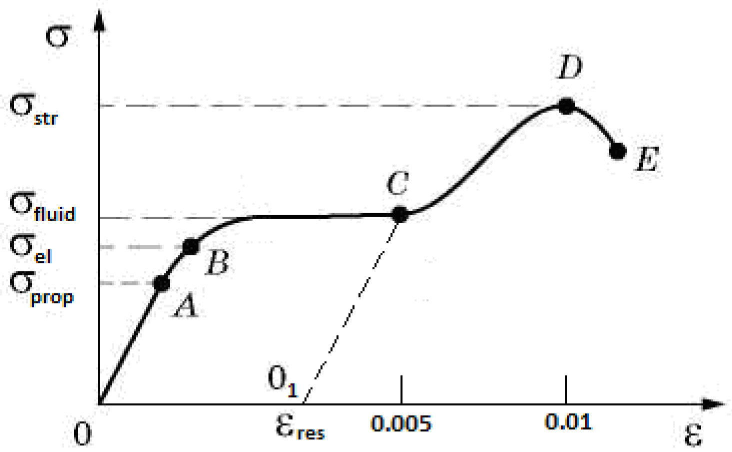


Fig. 1. The deformation diagram for mechanical strain of steel sample.

This curve can be conventionally divided into five zones.

The *OA* zone is called the zone of proportionality. Within this zone, Hooke's law holds.

The *OB* zone is the zone of elasticity, because the body will regain its size and shape if the stress relief in this zone of deformation.

The *BC* zone is called the zone of general fluidity. In this zone, the elongation of the sample occurs without a noticeable stress increasing.

The *CD* zone is the fixation zone, in this zone the elongation of the sample is accompanied by stress increasing, The places of future (the necks) destruction appear in the sample. Their formation (point D) is accompanied by the process of local fluidity in the zone *DE* and the following destruction of the sample.

If the stress is reduced in the *BC* zone, then the corresponding dependence $\sigma=f(\varepsilon)$ will run parallel to the line *OA* and will intersect the abscissa axis at some point O_1 (segment CO_1). The segment OO_1 determines the residual deformation ε_{res}, which characterizes the plastic deformation of the sample. Obtaining a deformation diagram allows one to determine a number of the most important characteristics and their corresponding values:

• the **limit of proportionality** σ_{prop} - the highest stress value at which Hooke's law is still fulfilled;

• the **elastic limit** σ_{el} - the highest stress value at which there are no residual deformations;

• the **fluidity limit** σ_{fluid} - the highest stress value at which there is an increase of deformation without a significant increase of the stress;

• the **strength limit** σ_{str} - the highest stress value that the sample can withstand.

Viscoelastic properties are often manifesting during the deformation of bodies. It happens because the stress depends not only on the deformation ε, but also on the rate of its change with time, ie the derivative $d\varepsilon/dt$.

Experimental results show that the strain or compression deformation diagrams differ from the one shown in figure 1 for most biological tissues. Usually the zone of the general fluidity is not expressed for biological materials, though this property is clearly exhibiting during the period of their functioning. The destruction of the material also occurs without noticeable stress decreasing, which takes place for the *DE* zone.

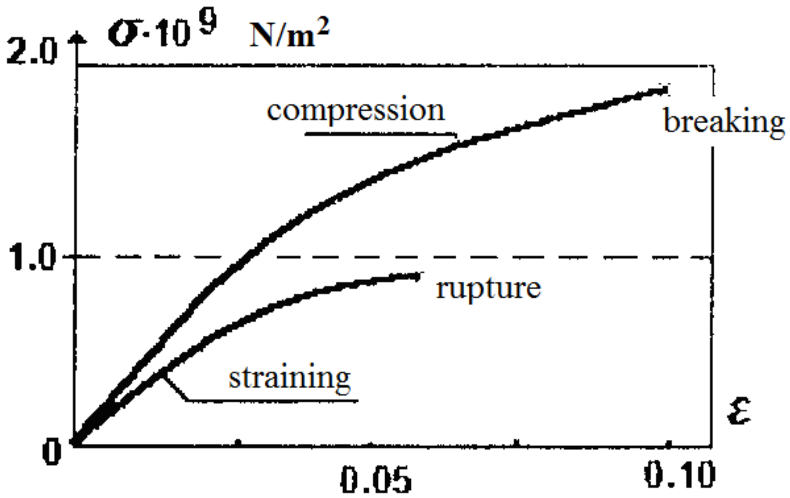


Fig. 2. The deformation diagram for bone.

Let us consider the deformation of a bone tissue. In general, we should note that the structure of bone tissue is quite complex. It is a composite material consisting of organic and inorganic substances, and exhibits anisotropic properties (ie its properties depend on the direction). The bone tissue is similar in its mechanical properties to wood, concrete and some metals, ie to the materials used in construction works.

The diagram of straining and compression along the longitudinal axis of the samples that were cut from the femur bone is shown in figure 2.

As one can see, the deformation of the bone occurs within large limits (comparing with the metal rod) - up to 10% during the compression and up to 5% during the straining. If the deformations are small (less than 2%), the bone behaves like a "Hook body", for which the dependence $\sigma=f(\varepsilon)$ is close to linear. It should be noted that the bone "works" better for compression than for straining. The strength limit and the size of deformations for compression are almost twice as large as those observed during the straining.

The following types of deformation are distinguished in mechanics (Fig. 3):

a) **volume deformation**, which occurs when the uniform distribution of compressive or tensile forces on the body surface;

b) **shear deformation**, manifested in the relative parallel displacement of the layers of the body relative to each other;

c) **torsion deformation** occurs in the sample, when one of its sections remains stationary, and in the other there is a pair of forces, the moment of which is directed along the axis of the sample;

d) **bending deformation**.

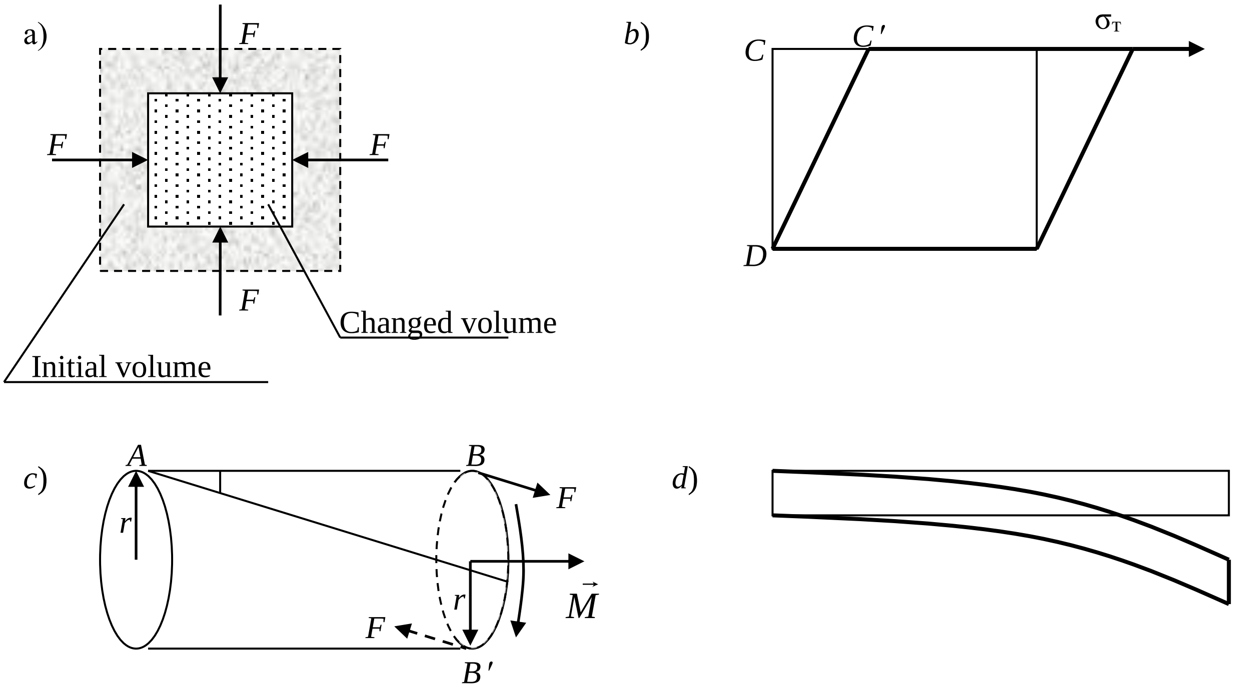


Fig. 3. Types of deformation: a) volume deformation, b) shear deformation, c) torsion deformation, d) bending deformation

Let us consider the bending deformation more detailed. The shift of the middle of the straight elastic rod $h = O_1O_2$ is taken as the value of the deformation in the case of the bending. The shift h obtained by the middle of the rod is called the bending deflection (Fig. 4).

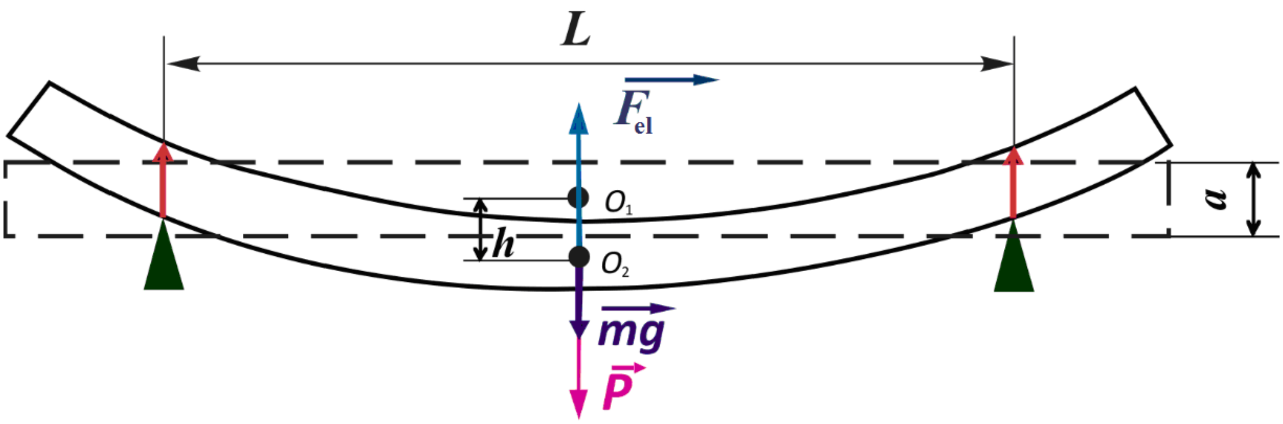


Fig. 4. The scheme of the bending deformation.

The bending deflection is bigger for the greater stress and it depends on the shape and size of the rod, as well as on the elastic modulus of the material.

If the force $\vec{P}$ is applied to the middle of a rod of length L, width b, and thickness a (see Fig. 4), then the bending deflection h will be determined according to the formula:

$$h = \frac{PL^3}{4ba^3E},$$

which gives

$$E = \frac{PL^3}{4ba^3h} \tag{2.7}$$

It is proved in the theory of elasticity that all types of deformation, including bending, can be reduced to the diagram of straining (compression) and bending. Indeed, the material is straining on the convex side (Fig. 5), and simultaneously there is compression on the concave side. Moreover, the straining and compression are less for layers which are closer to the middle layer *KN*.

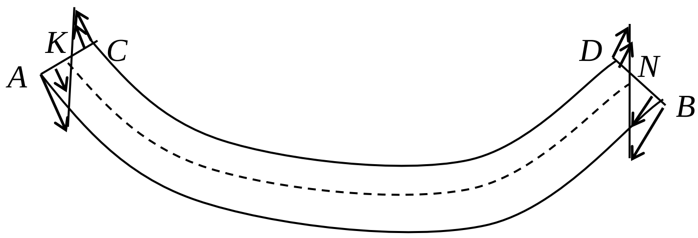


Fig. 5. Elastic forces during the bending deformation.

The *KN* layer is called neutral because it is not strained or compressed, (it is only distorted). Since the layers *AB* and *CD* undergo the greatest deformation of straining and compression, they have the greatest forces of elasticity (elastic forces in Fig. 5 are shown by arrows). These forces decrease from the outer layer to the neutral one. The inner layer does not undergo noticeable deformations and does not counteract external forces. Therefore the neutral layer is "inactive" in the interpretation of deformation. This explains the tubularity of the bones, which provides strength and lightness of their "construction".

Experimental facility.

The facility for determining of the elastic modulus consists of a massive platform 1 with two racks. The sample 2 is placed between these racks (see Fig. 6). There is a hook for hanging the loads 4 in the middle of the sample.

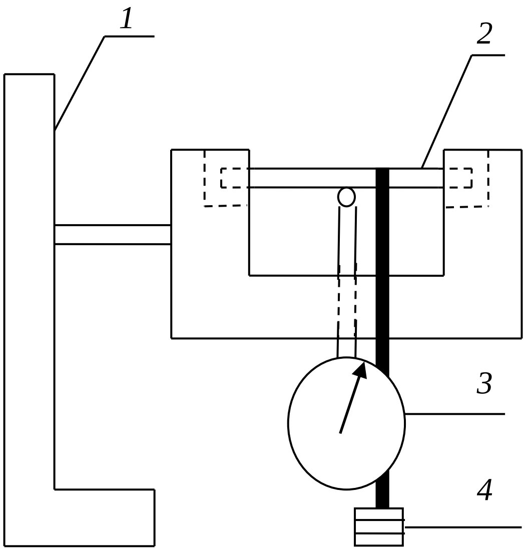


Fig. 6. The experimental facility.

The micrometer 3 is mounted on the rod in a vertical position. The rod is placed between the racks. The micrometer tip is applied to the middle of the sample. The micrometer hand deviates during contact with the contact plate when the sample is loaded (see Fig. 6).

Devices.

Various indicators are used to measure linear dimensions, determine the magnitude of deviations from a given geometric shape or the relative position of surfaces. The most famous of them are callipers and micrometers.

Vernier calliper is a measuring tool for measuring linear dimensions, outer and inner diameters, length, thickness, depth, etc (Fig.7).

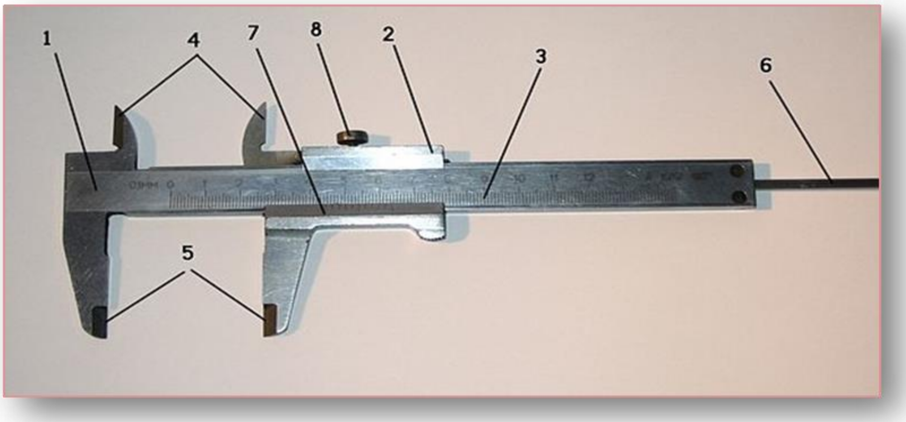


Fig. 7. The calliper brand ШЦ-1 with vernier, which has the following components: 1. movable beam; 2. frame; 3. scale of the beam; 4. grips for internal measurements; 5. grips for external measurements; 6. depth gauge ruler; 7. vernier; 8. frame fixing screw.

The movable beam is a measuring ruler with the minimal gradation of 1 mm. It has the length of 150 mm. Thus, the maximum size of the measured surface should not exceed 15 cm. But there are also calipers with longer beams. The frame of the caliper is movable and it is designed to move the grips. The grips can be expanded or narrowed to the required size moving the frame. The frame can be fixed with the screw (item 8). This is necessary for the tool does not lose the obtained position of the frame after measuring. The vernier is an auxiliary scale on the calliper. The vernier has 10 divisions with a size of 1.9 mm. Thus, the total length of the scale is 19 cm. The auxiliary scale is used to measure the size with an accuracy of 0.1 mm.

The measurement procedure using a vernier calliper. The value of the measurements determines the relative position of both scales: the main and the vernier. The surface of the vernier scale has a bevel for a better combination with the main scale.

Firstly, estimate the integer number of mm on the main scale, which is located to the left of the initial vernier mark. For example, if the zero point of the vernier stopped between the divisions of 23 mm and 24 mm, then the integer number of mm is 23. Next, determine the number of tenth parts of a unit. To do this, find a mark on the vernier scale, which clearly coincides with any mark on the main scale. The exact matching of these marks is very important! The value of the matching mark of the vernier gives number of tenths of a mm. If there are several such coincidences, then one must take into account the mark that is closer to the zero point of the vernier. For example (see fig. 8), there are two coinciding marks on a vernier scale: 6 and 7. In this case one must choose value "6".

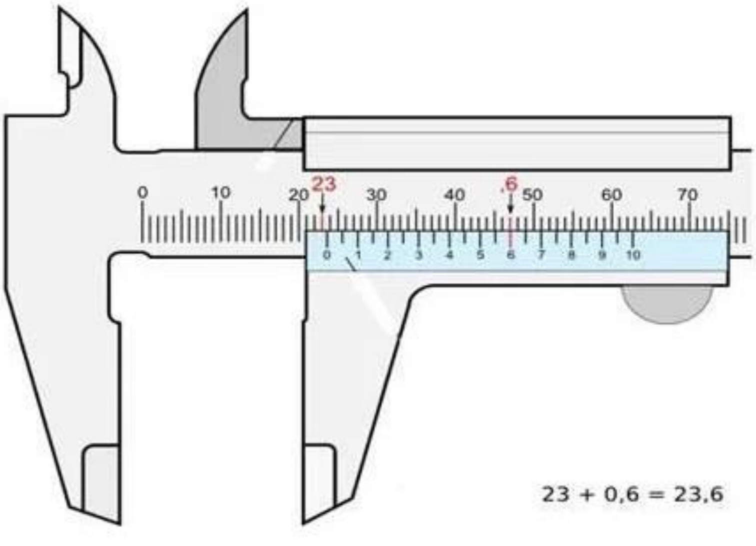


Fig. 8. Example of the measurement procedure using a vernier calliper

Experimenter can obtain the result by adding the integer number of a mm and number tenths of a mm. Therefore, one will obtain the total measurement result: 23.6 mm in the described example (see fig. 8).

The digital callipers are widely used at the present time. Such type of calliper gives the opportunity to obtain values in different units of measurement. The measurements also can be made with higher accuracy.

Fig. 9. The digital calliper.

The example of the digital calliper is shown in figure 9.

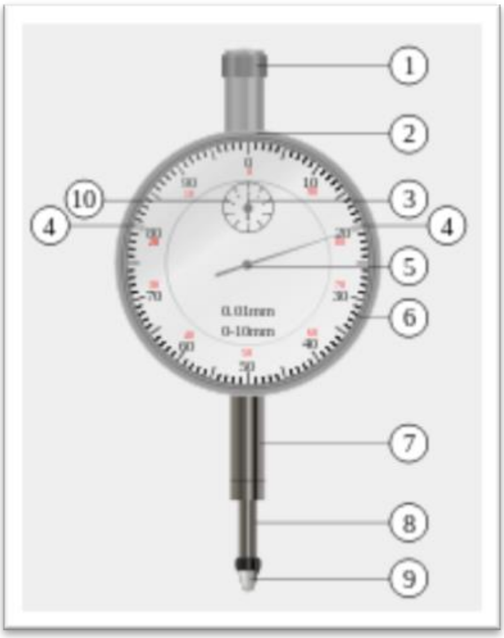
The results of measurements are directly shown in the digital display of the calliper.

Clock-type micrometer (in the technical literature it is called a dial indicator) is a device for measuring linear dimensions, in which the scale has a circular shape (dial shape). The measured value is marked by the hand (hands) on this dial scale (Fig. 10).

Fig. 10. The clock-type micrometer

Dial indicators are most often equipped with a rail-gear transmission, due to which the hand can perform several revolutions in the measuring range.

The design of the clock-type micrometer is presented in figure 11. It consists of the measuring head with the measuring tip 9 which can perform a longitudinal movement. The basis of the indicator is the housing 2, inside which a converting mechanism (rail-gear transmission) is mounted. A measuring rod 8 with a tip 9 passes through the housing in the sleeve 7. A rack is cut on the measuring rod. The movements of the measuring rod-rack 8 are transmitted by the gears to the main hand 5. The amount of rotation of the main hand 5 is shown on a dial scale 4. The round scale is rotated by the corrugated ring 6 to set the "0".



The measuring range of such indicator is from 0 to 10 mm. When setting the scale to zero division, the scale 4 of the indicator together with the corrugated ring 6 rotates relatively to the main hand 5.

The measurement procedure using a clock-type micrometer. There are two hands and two scales on the front of the indicator dial. The main (large) hand 5 is on the numerical circular scale 6 and the small hand 3 is on the counting (small) scale 10. The circular scale of the indicator has a division value of 0.01 mm, and the counting scale has a division value of 1 mm. The 1 mm movement of the measuring rod 8 causes the rotation of the hand 5 per 100 divisions (one full revolution), and the hand 3 per one division.

The permissible measurement error starts from 0.5 of division for the range of ± 10 divisions and is 2 divisions for the full range of measurements.

The measurement procedure.

1. Measure the width *b* and the thickness *a* of the samples using a calliper (the method of measuring using a calliper is given above).
2. The length *L* of the samples should be measured as the distance between the supporting points.
3. The measurements of geometric dimensions should be done three times for each sample in different places of the sample. Thus the values a_1, a_2, a_3; b_1, b_2, b_3; L_1, L_2, L_3 will be obtained for each of the three samples.
4. Calculate the average values $\langle b \rangle$, $\langle a \rangle$, $\langle L \rangle$ for each sample.
5. The results of measurements and calculations must be entered in the first line of table 1.

Table 1.

Sample	a_1, m	a_2, m	a_3, m	$\langle a \rangle$, m	b_1, m	b_2, m	b_3, m	$\langle b \rangle$, m	L_1, m	L_2, m	L_3, m	$\langle L \rangle$, m
1												
2												
3												

6. Put the sample 1 on the supports.
7. Install the clock-type micrometer so that the measuring tip of the micrometer touches the surface of the sample. Fix this position of the tip.
8. Rotate the corrugated ring (item 6, fig. 11) clockwise or counterclockwise to set the main and auxiliary hands of the micrometer to "zero".
9. Load the sample with loads *P* and take the appropriate micrometer readings, which are the bending deflection *h*.
10. Load the sample with loads of different masses *m* sequentially and write the corresponding values of the bending deflection for each load $P = mg$ to the table 2.
11. Calculate P/h for each load and then calculate the elastic modulus *E* for each of the three loads.
12. Calculate the average value $\langle E \rangle$. Calculate the error of the measurements ΔE.
13. The results of measurements and calculations are entered in table. 2.

Table 2 (for sample 1)

Num. of measur.	*m*, *kg*	*P*, N	*h*, m	*P/h*, N/m	*E*, Pa	$\langle E \rangle$, Pa	ΔE, Pa
1.							
2.							
3.							

14. Perform the same measurements and calculations for two samples of other materials.

Table 2 (for sample 2)

Num. of measur.	*m*, kg	*P*, N	*h*, m	*P*/*h*, N/m	*E*, Pa	<*E*>, Pa	Δ*E*, Pa
1.							
2.							
3.							

Table 2 (for sample 3)

Num. of measur.	*m*, kg	*P*, N	*h*, m	*P*/*h*, N/m	*E*, Pa	<*E*>, Pa	Δ*E*, Pa
1.							
2.							
3.							

15. Compare Young's modules of different materials. Make the conclusions based on the results of experiments.

Control questions.

1. Define the basic mechanical properties of the tissue.
2. What is the force of elasticity?
3. What is deformation?
4. What types of deformation do you know? Illustrate the answer.
5. Explain the nature of the force of elasticity.
6. What values characterize the deformation quantitatively?
7. What is mechanical stress?
8. What does the relative deformation show?
9. Formulate Hooke's law.
10. Explain the physical meaning of Young's modulus.
11. Draw a diagram of the straining of steel and explain all its areas.
12. Define the main characteristic points of this diagram.
13. What is the bending deflection?
14. Explain how the tubularity of a bone defines its properties.

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